290 A HW #1 Solutions

$\boxed{1}$ $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 10$ $\quad (1\times 3)\times(3\times 1)$ +1

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 8 & 10 \\ 12 & 15 \end{bmatrix}$ $\quad (3\times 1)\times(1\times 2)$ +1

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 11 & -11 \end{bmatrix}$ +1

$(2\times 3)\times(3\times 2)$

$\boxed{2}$ The above, in MATLAB +3

$\boxed{3}$ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} -1 & 3 \\ -1 & 7 \end{bmatrix}$ $\quad BA = \begin{bmatrix} 4 & 6 \\ 2 & 2 \end{bmatrix}$ +2

$AB \neq BA$

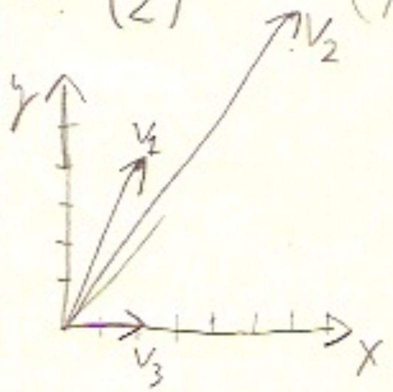
$A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 16 & -8 \\ -8 & 16 \end{bmatrix}$ $\quad BA = \begin{bmatrix} 16 & -8 \\ -8 & 16 \end{bmatrix}$ +2

$AB = BA$

4 $X = \{V_1, V_2, V_3\}$

$V_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $V_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $V_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



linearly dependent +2

$C_1 V_1 + C_2 V_2 = V_3$ +1

$C_1 = -2 \quad C_2 = 1$

$V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $V_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$C_1 V_1 + C_2 V_2 = V_3$ +1

linearly independent +2

5 $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}$ +2 for $p = 1, 2, \infty$

$V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $V_2 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ $V_1 \cdot V_2 = 23$

$7.07 \cdot 3.74 \cdot \cos\theta = 23$ +2

$\cos\theta = 0.869 \quad \theta = 29.6°$

$p=1 \quad \|V_1\|_1 = 6 \quad \|V_2\|_1 = 12$

$p=2 \quad \|V_1\|_2 = 3.74 \quad \|V_2\|_2 = 7.07$ +6

$p=\infty \quad \|V_1\|_\infty = 3 \quad \|V_2\|_\infty = 5$

$V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $V_2 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ $V_1 \cdot V_2 = \varnothing$

orthogonal +2

$p=1 \quad \|V_1\|_1 = 6 \quad \|V_2\|_1 = 3$

$p=2 \quad \|V_1\|_2 = 3.74 \quad \|V_2\|_2 = 1.73$ +6

$p=\infty \quad \|V_1\|_\infty = 3 \quad \|V_2\|_\infty =$ +1

$\boxed{6}$ In code, attached

$\boxed{7}$ If two vectors are orthogonal $(x \perp y)$

$$\langle x, y \rangle = \emptyset$$

$$x_1 y_1 + x_2 y_2 + \dots + x_n y_n = \emptyset$$

If these vectors are linearly dependent, then

$$x = cy \text{ for } c \neq \emptyset$$

$$\downarrow$$

$$x_i = c y_i$$

Plug into dot product equation

$$\emptyset = \sum_1^n c y_i^2$$ only possible if $y = \emptyset$ or $c = \emptyset$.

MAE 290A HW1 Problem 6

```
x1 = [1 ; 1.001];
x2 = [1 ; 1];
A = [1.1 -1;
    -1 1.1];

for k=1:20
    x1 = A*x1;
    x2 = A*x2;
end
x1
x2

%comment:  small changes in the input vector result in enormous
%differences after several iterations.
```

```
x1 =

   1.0e+03 *

   -1.3911
    1.3911


x2 =

   1.0e-19 *

    0.1000
    0.1000
```