

HOMEWORK #2 - WILL NOT BE GRADED

1. Compute by hand the LU Decomposition, the LU Decomposition with Partial Pivoting and the LU Decomposition with Complete Pivoting of the matrices:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}.$$

2. Based on the LU Decompositions you computed in P. 1, what is the rank of matrix A ? Justify your answer.
3. Use the Matlab function `lu` to compute the LU Decomposition of the matrices in P.1. Which form of the decomposition did Matlab perform?
4. Prove that if a matrix A has the LU Decomposition $A = LU$ then its determinant is equal to the product of the diagonal entries of U . Use this fact to compute the determinant of A in P. 1.
5. Use the LU Decomposition to classify the set of vectors $X = \{v_1, v_2, v_3\}$

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

as linearly dependent or linearly independent.

6. Solve the linear system

$$Ax = b, \quad A = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 5 & 0 \\ 3 & 6 & 0 \end{bmatrix}, \quad b = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

in Matlab by explicitly computing the LU Decomposition of A then solving two triangular systems of equations. Compare the accuracy and the time it took to compute your solution with solutions obtained with a) the slash operator (`\`); b) the function `inv`.

7. Generate a random 500×500 A matrix and a 500×1 b vector in Matlab using `randn`. Consider the linear system

$$AAx = b$$

Identify (and explain why) which of the Matlab expressions are equivalent ways to compute the solution:

- (a) `C = inv(A); x = C * C * b;`
- (b) `x = A \ A \ b;`
- (c) `x = A \ (A \ b);`
- (d) `x = (A * A) \ b;`
- (e) `[L,U]=lu(A); x = U \ (L \ (U \ (L \ b)));`

Verify your conclusions in Matlab. Time each computation and explain the times you obtained.