HOMEWORK #3

1. Use Matlab to compute the QR Decomposition $A = QR$ for the matrices

$$A_1 = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

2. A QR Decomposition with column-pivoting produces a decomposition of the form $AE = QR$, where $Q^T Q = I$, R is upper-triangular and E is a permutation matrix. Explain how E can be chosen so as to produce R where the diagonal entries appear in decreasing absolute value order. Use Matlab to compute the pivoted QR decomposition $AE = QR$ for the matrices in P.1.
Hint: Use the orthogonality of the intermediate rotations, e.g. Householder reflectors H_i's!

3. Use the QR decompositions you computed in P.1 or P.2 to calculate the "least-square solution" to the linear systems $A_i x_i = b_i$, $i = 1, \ldots, 4$, where A_i, $i = 1, 2, 3$, are as in P.1 and

$$b_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad b_2 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \qquad b_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad A_4 = A_2, \qquad b_4 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}.$$

For each problem, compute the norm of the residual $\|A_i x_i - b_i\|$, $i = 1, \ldots, 4$, and explain whether the equation $A_i x_i = b_i$ has a solution or not and if this solution is unique.

4. Compute by hand the eigenvalues and eigenvectors of the matrices

$$A_1 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 0 & 1 \\ -8 & 6 \end{pmatrix}, \qquad A_3 = \begin{pmatrix} 0 & 1 \\ -16 & 8 \end{pmatrix}.$$

When possible, use the eigenvalues and eigenvectors to construct an eigenvalue decomposition of the form $A = X \Lambda X^{-1}$, where Λ is a diagonal matrix. When not possible, explain what goes wrong. Repeat your calculations in Matlab.

5. One of your friends claimed that if a non-symmetric matrix has repeated eigenvalues then it must be defective. Prove your friend wrong by computing the eigenvalue decomposition of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

by hand. Repeat your calculations in Matlab.

Notes:

1) Clearly justfiy all your answers!

2) You may use Matlab/Octave for calculations in all questions except when noted that you should perform the calculation by hand.