MAE 290 A HW 3 Solutions

1) See MATLAB code & output

2) E (the permutation matrix) can be constructed by swapping columns in A so that the 2-norms of each column decreases across the permutated matrix. This results in decreasing values along the diagonal of R after QR decomposition.

3) See MATLAB code & output

A1 & b1 – Full rank in R allows for unique solution, small normed error implies least squares solution is accurate

A2 & b2 – Full rank in R allows for unique solution, large normed error implies that no solution exists for $A_2 x = b_2$.

A_3 & b3 – R not having full rank implies there is not a unique solution. The small error implies that the calculated solution is good

A_2 & b_4 — Full rank in R allows for unique solution, small normed error implies solution is accurate.

4) $A_1 = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ $A_2 = \begin{bmatrix} 0 & 1 \\ -8 & 6 \end{bmatrix}$ $A_3 = \begin{bmatrix} 0 & 1 \\ -16 & 8 \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\lambda_\pm = \frac{1}{2}\left[(a+d) \pm \sqrt{4bc + (a-d)^2}\right]$

if $b \neq \emptyset$ $s_\pm = \begin{bmatrix} b \\ \lambda_\pm - a \end{bmatrix}$

if $c \neq \emptyset$ $s_\pm = \begin{bmatrix} \lambda_\pm - d \\ c \end{bmatrix}$

if $b = c = \emptyset$, $\lambda_+ = a$ $\lambda_- = d$

$s_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $s_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

A_1: $\lambda_\pm = \frac{1}{2}\left[(3+3) \pm \sqrt{4+\emptyset}\right]$

$\lambda_+ = 4$ $s_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $s_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda_- = 2$

$A_2: \lambda_\pm = \frac{1}{2}\left[(\emptyset+6) \pm \sqrt{(4\cdot -8) + (-6)^2}\right]$

$$\lambda_\pm = \frac{1}{2}\left(6 \pm \sqrt{4}\right)$$

$$\lambda_+ = 4 \quad \lambda_- = 2$$

$$S_\pm = \begin{bmatrix} \underline{1} \\ 4 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$A_3: \lambda_\pm = \frac{1}{2}\left[(\emptyset+8) \pm \sqrt{(4\cdot -16) + (-8)^2}\right]$

$$= \frac{1}{2}\left(8 \pm \sqrt{\emptyset}\right)$$

$$\lambda_1 = 4 = \lambda_2$$

$S_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix} = S_2$ matrix is defective (linearly dependent eigen vectors)

Eigenvalue decomposition:

$$X_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad X_1^{-1} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \quad X_2^{-1} = \begin{bmatrix} -1 & 0.5 \\ 2 & -0.5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0.5 \\ 2 & -0.5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -8 & 6 \end{bmatrix} = A_3$$

No eigen-decomposition for A_3.
Cannot invert eigenvector matrix.

5) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ Find eigen decomposition

$$|\lambda I - A| = \emptyset$$

$$\left| \begin{bmatrix} \lambda-1 & 0 & -1 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-2 \end{bmatrix} \right| = \emptyset$$

$(\lambda-1)(\lambda-1)(\lambda-2) = \emptyset \qquad \lambda = 1, 1, 2$

$-1\,[0 - (\lambda-1)] = \emptyset$

$\lambda = 1$

$(\lambda I - A)s = \emptyset$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} s = \emptyset \qquad s_{1,2} = \begin{bmatrix} x_1 \\ x_2 \\ \emptyset \end{bmatrix}$$

linearly independent

$\lambda = 2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} s = \emptyset \qquad s_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Matrix is not defective.

MAE 290A Homework 3 Solutions

Contents

Problem 1

```
A1 = [1 3; 2 4];
A2 = [1 4; 2 5; 3 6];
A3 = [1 2 3; 4 5 6];

[Q1, R1] = qr(A1)
[Q2, R2] = qr(A2)
[Q3, R3] = qr(A3)
```

```
Q1 =

   -0.4472   -0.8944
   -0.8944    0.4472


R1 =

   -2.2361   -4.9193
         0   -0.8944


Q2 =

   -0.2673    0.8729    0.4082
   -0.5345    0.2182   -0.8165
   -0.8018   -0.4364    0.4082


R2 =

   -3.7417   -8.5524
         0    1.9640
         0         0


Q3 =

   -0.2425   -0.9701
   -0.9701    0.2425


R3 =
```

```
   -4.1231   -5.3358   -6.5485
         0   -0.7276   -1.4552
```

Problem 2

```
[Q1a, R1a, E1] = qr(A1)
[Q2a, R2a, E2] = qr(A2)
[Q3a, R3a, E3] = qr(A3)
```

```
Q1a =

   -0.6000   -0.8000
   -0.8000    0.6000


R1a =

   -5.0000   -2.2000
         0    0.4000


E1 =

     0     1
     1     0


Q2a =

   -0.4558    0.7909    0.4082
   -0.5698    0.0930   -0.8165
   -0.6838   -0.6048    0.4082


R2a =

   -8.7750   -3.6467
         0   -0.8374
         0         0


E2 =

     0     1
     1     0


Q3a =

   -0.4472   -0.8944
   -0.8944    0.4472
```

```
R3a =

   -6.7082   -4.0249   -5.3666
         0    0.8944    0.4472


E3 =

     0     1     0
     0     0     1
     1     0     0
```

Problem 3

```
b1 = [1; -1];
b2 = [1; 2; 4];
b3 = b1;
b4 = [3; 3; 3];

X1 = R1\(Q1'*b1)
X2 = R2\(Q2'*b2)
X3 = R3\(Q3'*b3)
X4 = R2\(Q2'*b4)

n1 = norm(A1*X1-b1,2)
n2 = norm(A2*X2-b2,2)
n3 = norm(A3*X3-b3,2)
n4 = norm(A2*X4-b4,2)
```

```
X1 =

   -3.5000
    1.5000


X2 =

    1.7222
   -0.2222


X3 =

   -1.5000
         0
    0.8333


X4 =

   -1.0000
    1.0000
```

```
n1 =

   2.3915e-15


n2 =

    0.4082


n3 =

   2.2204e-15


n4 =

   1.3323e-15
```

Problem 4

```
A1 = [3 1; 1 3];
A2 = [0 1; -8 6];
A3 = [0 1; -16 8];
[V1, D1] = eig(A1)
[V2, D2] = eig(A2)
[V3, D3] = eig(A3)
```

```
V1 =

   -0.7071    0.7071
    0.7071    0.7071


D1 =

     2     0
     0     4


V2 =

   -0.4472   -0.2425
   -0.8944   -0.9701


D2 =

     2     0
     0     4
```

```
V3 =

    0.2425   -0.2425
    0.9701   -0.9701


D3 =

     4     0
     0     4
```

Problem 5

```
A = [1 0 1; 0 1 0; 0 0 2];
[V, D] = eig(A)
```

```
V =

    1.0000         0    0.7071
         0    1.0000         0
         0         0    0.7071


D =

     1     0     0
     0     1     0
     0     0     2
```